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1.Relative humidity definitions:

Relative humidity is at all temperatures and pressures defined as the ratio of the water vapour pressure to the saturation water vapour pressure (over water) at the gas temperature:

$$RH = P_W/P_{WS} * 100\% \tag{1}$$

The total pressure does not enter the definition. Above 100°C the same definition is valid. But as the saturation vapour pressure P_{WS} is greater than 1013 hPa (normal ambient pressure) the RH can´t reach 100% in an unpressurised system.

Below 0°C the definition is also valid. Here 100%RH is also impossible because condensation will occur at a lower humidity than 100% (when the vapour is saturated against <u>ice</u>)

2. Water vapour saturation pressure:

The following formula gives the water vapour saturation pressure to sufficient accuracy between 0°C and 200°C (Hyland,R.,Wexler,A.:"Formulations of the Thermodynamic Properties of the Saturated Phases of H2O from 173.15 K to 473.15 K",Ashrae transactions 1983, Part 2A. pp 500-513):

1. Calculate the virtual temperature:

$$\Theta = T - \sum_{i=0}^{3} C_i T^i \quad \text{,where} \tag{2}$$

T=temperature in K
C_i=coefficients,
C_0=0.4931358
C_1=-0.46094296*10^{-2}
C_2=0.13746454*10^{-4}
C_3=-0.12743214*10^{-7}

2. Calculate the vapour pressure :

$$\ln P_{ws} = \sum_{i=-1}^{3} b_i \cdot \Theta^i + b_4 \cdot \ln \Theta \qquad \text{(Pa)} \qquad \text{,where}$$
(3)

b_i=coefficients,
b_{-1}=-0.58002206*10^4
b_0=0.13914993*10^1
b_1=-0.48640239*10^{-1}
b_2=0.41764768*10^{-4}
b_3=-0.14452093*10^{-7}
b_4=6.5459673

Correspondingly the saturation water vapour pressure over **ice** at temperatures between -100 and 0.01°C can be calculated using:

$$\ln P_{wi} = \sum_{i=0}^{5} a_i \cdot T^{i-1} + a_6 \cdot \ln T \quad \text{(Pa)} \qquad \text{,where} \qquad (4)$$

a_i=coefficients,
a_0 = -5674.5359
a_1 = 6.3925247
a_2 = -9.677843*10^{-3}
a_3 = 6.2215701*10^{-7}
a_4 = 2.0747825*10^{-9}
a_5 = -9.484024*10^{-13}
a_6 = 4.1635019

If lower accuracy can be tolerated a simpler formula can be used for the water vapour saturation pressure over water (and over ice):

$$P_{ws}=A*10^{(m*T/(T+Tn))} \qquad \text{(hPa)} \quad \text{,where} \qquad (5)$$

A,m,T_n=constants see table 1
T=temperature (°C)

Temperature range(°C)	A	m	Tn	max error
over water:				
0...60	6.1078	7.5	237.3	0.06%
0....200	6.0964	7.33354	230.5	0.38%
-20 ... 50	6.1162	7.5892	240.71	0.09%
over ice:				
-70...0	6.1134	9.7911	273.47	0.05%

Table 1. Constants for formula 5.

(see paragraph 9 for information on the enhancement factor for the saturation vapour pressure)

3. Calculation of dewpoint from RH:

Calculate P_{ws} using formula (3) or (5)

Calculate P_w=Pws*RH/100 (in hPa!)

Calculate Dewpoint using formula (6):

$$Td = \frac{T_n}{\left[\frac{m}{^{10}\log\left(\frac{Pw}{A}\right)} - 1 \right]}$$

(6)

The constants in formula (6) are the same as for formula (5).

Example:

The ambient temperature is 40 °C and the RH is 50%. Calculate T_d:

Pw=P_{ws}(40°C)*50/100=36.88 hPa

T_d=237.3/(7.5/^{10}log(36.88/6.1078)-1)=<u>27.6 °C</u>

3.1 Calculation of dewpoint at different pressure

1. Calculate Pws using formula (3) or (5)
2. Calculate measured vapor pressure (at measurement pressure

$$Pw_{meas} = Pws \cdot \frac{RH}{100}$$

3. Multiply with pressure ratio:

$$Pw_{pres} = \frac{P_{pres}}{P_{meas}} \cdot Pw_{meas}$$

4. Calculate Dewpoint at process pressure:

$$Td_{pres} = \frac{T_n}{\left[\frac{m}{^{10}\log\left(\frac{Pw_{pres}}{A}\right)} - 1 \right]}$$

3.2 Calculation of RH from dewpoint and ambient temperature

$$RH = 100\% \cdot \frac{Pws(Td)}{Pws(Tambient)}$$

Pws may be calculated using formulas (3) or (5). If the simplified formula (5) is used the expression for RH may be further simplified to:

$$\left[\frac{Td}{} - \frac{Tambient}{} \right]$$

4. Using psychrometers:

The drybulb-temperature T_{dry} and the wetbulb-temperature T_{wet} can be converted into P_w using formula (7):

$$P_w = P_{ws}(T_{wet}) - P_{tot}*K*(T_{dry} - T_{wet}) \qquad \text{,where} \qquad (7)$$

P_{ws}= water vapour saturation pressure from formula (6)
P_{tot}= total ambient pressure
K=psychrometer constant 0.000662 $°C^{-1}$

When P_w is known RH can be calculated using (1) or T_d can be calculated using (6)

Example:

T_{wet} is 38.5°C, T_{dry}=40.0°C and the ambient pressure is 1013 hPa .Calculate RH and T_d:

P_{ws}(38.5°C)=68.05 hPa
P_{ws}(40.0°C)=73.75 hPa

P_w=68.05 - 1013*0.000662*(40.0-38.5)=67.04 (hPa)

RH=67.04/73.75*100 =90.9%

T_d=237.3/(7.5/^{10}log(67.04/6.1078)-1=<u>38.22 °C</u>

5. Mixing ratio:

The mixing ratio (mass of water vapour/mass of dry gas) is calculated using (8):

$$X = B * P_w / (P_{tot} - P_w) \quad\quad [g/kg] \quad ,where$$
(8)

B=621.9907 g/kg

The value of B depends on the gas. 621.9907 g/kg is valid for air.

In general the constant can be calculated using:

$$B = M(h2o)/M(gas) * 1000 \quad\quad [g/kg] \quad ,where$$

M(h2o)=molecular weight of water
M(gas)=molecular weigth of gas

For instance for hydrogen we get

B=18.015/2.016*1000=8936 g/kg

Example:

The dewpoint T_d is 40°C and the total ambient pressure P_{tot} is 998 hPa. Calculate mixing ratio:

$P_w = P_{ws}(40°C) = 73.75$ hPa

$X = 621.9907 * 73.75/(998-73.75) = \underline{49.63 \text{ g/kg}}$

(To obtain the mixing ratio in units of grains/pound use B=4354 [grains/pound])

6. Enthalpy

Enthalpy can be calculated from mixing ratio using (9):

$$h = T*(1.01 + 0.00189X) + 2.5X \quad \text{(kJ/kg)} \quad \text{,where} \qquad (9)$$

T=temperature (°C)
X=mixing ratio (g/kg)

To convert to BTU/LB divide by 2.324
Example:

The ambient temperature is 20°C and the relative humidity is 50%.
Calculate enthalpy:

$Pw = P_{ws}(20°C)*50/100 = 11.69$ hPa

$X = 621.9907*11.69/(1013-11.69) = 7.26$ g/kg

$h = 20*(1.01 + 0.00189*7.26) + 2.5*7.26 = \underline{38.62}$ kJ/kg

7.Absolute humidity

Absolute humidity is defined as the mass of water vapour in a certain volume. If ideal gas behaviour is assumed the absolute humidity can be calculated using (10):

$$A = C \cdot P_W / T \qquad (g/m^3), \text{where} \qquad\qquad (10)$$

C=constant 216.679 gK/J
P_W=vapour pressure in hPa
T=temperature i K

Example:

The ambient temperature is 20°C and the relative humidity is 80%. Calculate absolute humidity:

$Pw = P_{WS}(20°C) \cdot 80/100 = 18.7$ hPa

$A = 216.679 \cdot 18.7/(273.16+20) = \underline{13.82 \ g/m^3}$

8. Parts per million (ppm)

Parts per million values are usually given vs. the amount of dry air:

I: Volume/volume PPM_v(dry):

$$PPM_v = \frac{P_w}{(P_{tot} - P_w)} 10^6 \qquad (11)$$

Where

P_w=water vapour pressure
P_{tot}=total pressure

II: Mass/mass PPM_m(dry)

$$PPM_m = \frac{M_w P_w}{M_d(P_{tot} - P_w)} 10^6$$
$$\frac{M_w}{M_d} = 0.62199 \qquad (12)$$

Where

P_w=water vapour pressure
P_{tot}=total pressure
M_w=molecular mass of water
M_d=molecular mass of dry air

From wet air:

III: Volume/volume PPM_v(wet):

$$PPM_v = \frac{P_w}{P_{tot}} 10^6 \qquad (13)$$

IV: Mass/mass PPM_m(wet)

$$PPM_m = \frac{M_w P_w}{M_d P_{tot}} 10^6$$
$$\frac{M_w}{M_d} = 0.62199 \qquad (14)$$

example:

The dewpoint is 7°C and the total pressure is 998 hPa calculate PPM_V(dry).

By using formula (4) we get P_{ws}(7°C)=10.02 hPa. Now

$$PPM_v = \frac{10.02}{(998 - 10.02)} 10^6 = 10142$$

9. Enhancement factor

The water vapour saturation pressures described in paragraph 2 are exactly valid only in vacuum (water vapour is the only gas present). If other gases are present the real saturation vapour pressure P_{ws} will increase. For ambient pressure and lower pressures this effect is typically ignored. But at pressures significantly above ambient it has to be taken into account. The enhancement factor f has been defined as follows:

$$f = \frac{x_w * P}{P_{ws}}$$

where,

x_w=the mole fraction of water vapour in the gas phase
P=total pressure
Pws=the saturation water vapour pressure (from paragraph 2)

x_w*P is in effect the water vapour pressure at saturation at pressure P

For CO_2-free air the following equation for f has been proposed (Greenspan:J.of Research of the NBS vol 80A, No. 1 p 41-44)

$$f = e^{\left[\alpha \left(1 - \frac{P_{ws}}{P} \right) + \beta \left(\frac{P}{P_{ws}} - 1 \right) \right]}$$

The parameters α and β depend on the temperature as follows:

$$\alpha = \sum_{i=1}^{4} A_i t^{(i-1)}$$

$$\beta = e^{\sum_{i=1}^{4} \left[B_i t^{(i-1)} \right]}$$

t=temperature in °C

the parameters A_i and B_i are listed below:

	water -50 to 0°C	water 0 to 100°C	ice -100 to 0°C
A1	3.62183×10^{-4}	3.53624×10^{-4}	3.64449×10^{-4}
A2	2.60553×10^{-5}	2.93228×10^{-5}	2.93631×10^{-5}
A3	3.86501×10^{-7}	2.61474×10^{-7}	4.88635×10^{-7}
A4	3.82449×10^{-9}	8.57538×10^{-9}	4.36543×10^{-9}
B1	-10.7604	-10.7588	-10.7271
B2	6.39725×10^{-2}	6.32529×10^{-2}	7.61989×10^{-2}
B3	-2.63416×10^{-4}	-2.53591×10^{-4}	-1.74771×10^{-4}
B4	1.67254×10^{-6}	6.33784×10^{-7}	2.46721×10^{-6}

The formulas above are fitted for the pressure range 1 ... 20 atm.

for instance at 20 °C and 10 bars f=1.0312